Random Algebraic Problems in '*Gaṇita Kaumudī* '

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Introduction

Our ancient Indian mathematicians had a sense of humour and were fun-loving. They used to make learning and doing mathematics interesting and fun-filled activity.

There are many unique problems in *Gaṇita Kaumudī* and this paper analyses a few of them from the 'Fractions' chapter. On fractions these types of problems are not often found in other mathematical texts. It will be a worthwhile attempt if those problems are brought to light.

***Bhinnaparikarman* (Fractions)**

'*Bhinnaparikarman*' is one of the important topics in our ancient Indian mathematical works. References can even be found in *Rg vēda, Maitrāyaṇī Saṁhitā* and *Ṣ̀ulba Sūtrā*. Indian mathematicians divided 'Fractions' into different classes (*Jāti)* according to the form.

Reduction of fractions to a common denominator is called '*Kalāsavarṇana'* meaning 'making (fractions) have the same colour. "Hindu mathematicians used to divide fractions into different classes, probably, according to the form in which these were used to be written and their nature. There is no uniformity about it'[1].

Nārāyaṇa Paṇḍita classified fractions into six categories. They are: (1) *Bhāgajāti*, (2) *Prabhāgajāti*, (3) *Bhāgānubandha*,(4) *Bhāgāpavāha,*(5) *Svāṁśānubandha* and (6) *Svāṁśāpavāha*.

'The need for the division of fractions into classes arose out of the lack of proper symbolism to indicate mathematical operations. The only operational symbol used by Hindus was a dot for the negative sign'[2].

Further, NārāyaṇaPaṇḍita, classified problems as equation of:

1. *Svarṇajāti*
2. *Ṣ̀eṣamūlajāti*
3. *Guṇamūlajāti*
4. *Hīnavargajāti*
5. *Amśavargajāti*
6. *Bhāgasaṃguṇyajāti*
7. *Bhinnasaṃdṛśyajāti*

This paper deals with the examples given under '*Ṣ̀eṣamūlajāti*' and a unique problem involving true or false statements.

[1] Shukla p.118
[2] HHM Vol.1 p. 188

Example - 1

कान्तायाःसुरतप्रसङ्गसमयेभिन्नाचमुक्तावली

मुक्तानांचपदद्वयंविचरणंशय्यापटस्योपरि।

तच्छेषस्यपदंत्रिभागयुगलेनाऽऽढ्यंप्रियेणाऽऽहृतं

तच्छेषस्यपदंक्षितौनिपतितंसूत्रेद्वयंकिंवद॥

- Chapter I (Example 40)

"During a love quarrel, the lady's necklace made up of pearls was broken. Twice the square-root less ¼ of the root of pearls were on the cover of the bed. The square-root of the rest along with 2/3 of this root were seized by the lover. The square-root of the rest fell down on the earth and 2 pearls were in the string of the garland. How many pearls were there in the garland?"

Total number of pearls = x

Pearls on the bed = $2\sqrt{x} - \frac{1}{4}\sqrt{x}$

= $x - \frac{7}{4}\sqrt{x}$ say 'a'

Seized by the lover = $\sqrt{a} + \frac{2}{3}\sqrt{a}$

Balance in the string $= a - \sqrt{a} - \frac{2}{3}\sqrt{a}$

= $a - \frac{5}{3}\sqrt{a}$ Let it be 'b'

Fallen on the floor = $\sqrt{b}$

Balance = $b - \sqrt{b}$

Balance in the string = 2

Therefore $b - \sqrt{b}$ = 2

i.e $b = 4$

-> $a - \frac{5}{3}\sqrt{a} = 4$

-> $3a - 5\sqrt{a} = 4$

solving this we get

a = $3^2 = 9$

-> $x - \frac{7}{4}\sqrt{x} = 9$ -> $4x - 7\sqrt{x} = 36$

solving this we get

-> $x = 4^2 = 16$

Total number of pearls in the garland is 16.

Example 2

गणेशंपद्मेनत्रिनयनहरिब्रह्मदिनपान्

विलोमैःशेषांशैर्विषयलवपूर्वैश्चकमलाम्।

पदेनाऽऽपूज्यैकेनचगुरुपदाम्भोजयुगलं

सरोजेनाऽऽचक्ष्वद्रुतमखिलमम्भोजनिचयम्॥ -

Chapter I (Example 41)

"Ganesa was worshipped with 1 lotus flower. Siva, Hari, Brahma and the Sun were worshipped with 1/5, ¼,1/3 and ½ of what remained, successively. Kamala was worshiped with the square-root of the lotus flowers and two feet of the teacher resembling lotus flowers were worshipped with 1 lotus flower. How many lotus flowers were there in the collection?"

Total Flowers = x

Ganesa = 1… (1)

Balance = $(x - 1)$ … (2)

Siva = $\frac{1}{5}(x - 1)$ … (3)

New Balance (2) – (3) = $\frac{4}{5}(x - 1)$…(4)

Vishnu $= \frac{1}{4} (\frac{4}{5} (x-1))$

$= \frac{1}{5} (x-1) \ldots (5)$

New Balance (4) – (5) $= \frac{3}{5} (x-1) \ldots (6)$

Brahma $= \frac{1}{3} (\frac{3}{5} (x-1))$

$= \frac{1}{5} (x-1) \ldots (7)$

New Balance (6) – (7) $= \frac{2}{5} (x-1) \quad \ldots \ (8)$

Sun $= \frac{1}{2} (\frac{2}{5} (x-1))$

$= \frac{1}{5} (x-1) \ldots (9)$

Kamala $= \sqrt{x}$

Teacher $= 1$

$1 + \frac{1}{5} (x-1) + \frac{1}{5} (x-1) + \frac{1}{5} (x-1) +$

$\frac{1}{5} (x-1) + \sqrt{x} + 1 \quad = x$

$2 + \frac{4}{5} (x-1) + \sqrt{x} = x$

$\sqrt{x} = x - 2 - \frac{4}{5} (x-1)$

$5\sqrt{x} = \frac{5x - 10 - 4x +}{5}$

$5\sqrt{x} = x - 6$

$25x = (x-6)^2$

$25x = x^2 - 12x + 36$

$0 = x^2 - 37x + 36$

$(x-1)(x-36) = 0$

$x = 1$ or 36

Answer cannot be 1 since Ganesa was worshipped with 1 flower, Teacher was worshipped with 1 flower etc.

Therefore, total number of flowers is 36.

Example 3

यातेनृपेमृगयुभिर्मृगयार्थमाशुपाशान्प्रसारयतित
त्त्रिलवोऽप्यटव्याम्।

शेषस्यघोरतरकेसरीपीडितानित्रीणिप्रचक्ष्वसचतु
ष्कपदानिविद्वन्॥

- Chapter I (Example 39)

"A king went out for hunting. The hunters laid the trap. There, 1/3 of the deer netted themselves. Thrice the square-root of the rest along with 4 were trouble by the ferocious lion. How many of them were there?"

Trapped Deer : $\frac{x}{3}$

Deer threatened by lion : $3 \text{ x } \sqrt{\frac{2}{3} x} + 4$

Therefore

$$x = \frac{x}{3} + 3 \sqrt{\frac{2}{3} x} + 4$$

$$\frac{2}{3} x - 3 \sqrt{\frac{2}{3} x} + 4$$

Let $y = \frac{2}{3} x$

$y - 3\sqrt{y} = 4$

$a = 1$, $b = -3$ and $c = 4$ and roots of the equation can be found using the formula

$$\sqrt{y} = \frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$$

$$\sqrt{y} = \frac{3 \pm \sqrt{3^2 + 4.1.4}}{2.1}$$

$\sqrt{y} = \frac{3 \pm \sqrt{25}}{2}$

$\sqrt{y} = \frac{3 \pm 5}{2}$

$\sqrt{y} = 4$ or -1

$y = 4^2 = 16$ (as per Rule 39)

Since $y = \frac{2}{3}x$, $x = 24$.

Solution : Total Deers = 24.

True or False statements problem

Nārāyaṇa Paṇḍita in Chapter II of *Gaṇita Kaumudī* discusses miscellaneous problems such as Partnership, Interest calculation,Gems, meeting of travellers etc. One of such problems is 'True or False' statements.

सत्यानृतेसूत्रम्।

सैकेष्टगुणाःपुरुषाद्विगुणेष्टानाभवन्त्यसत्यानि।
तैरूनापुरुषकृतिःशेषंसत्यानिवचनानि॥

- Chapter II (Rule 42(b) -43 (a))

"Add 1 to (the number of) persons liked (by a lady). Multiply (the sum) by the number of persons. (the product) less twice (the number of) persons liked is the number of false statements. Subtract the same from the square of the number of persons. The remainder is the number of true statements."

Let the lady like n persons out of m persons while others are disliked by her. Suppose that she speaks to each person that only he is liked, while others disliked.

Then the number of false statements

$= m(n+1)-2n,$

and

The number of True statements

$= m^2 - m(n+1) + 2n.$

This can be presented using a table :

	A1	A2	A3	….	An	An+1	An+2	An+3	….	Am	TRUE	FALSE
A1	T	F	F	F	F	T	T	T	T	T	*m-n+1*	*n-1*
A2	F	T	F	F	F	T	T	T	T	T	*m-n+1*	*n-1*
A3	F	F	T	F	F	T	T	T	T	T	*m-n+1*	*n-1*
….	F	F	F	T	F	T	T	T	T	T	*m-n+1*	*n-1*
An	F	F	F	F	T	T	T	T	T	T	*m-n+1*	*n-1*
An+1	F	F	F	F	F	F	T	T	T	T	*m-n-1*	*n+1*
An+2	F	F	F	F	F	T	F	T	T	T	*m-n-1*	*n+1*
An+3	F	F	F	F	F	T	T	F	T	T	*m-n-1*	*n+1*
….	F	F	F	F	F	T	T	T	F	T	*m-n-1*	*n+1*
Am	F	F	F	F	F	T	T	T	T	F	*m-n-1*	*n+1*

TRUE	$n(m-n+1)+(m-n)(m-n-1)$
	$m^2 - m(n+1) + 2n$
FALSE	$n(n-1) + (m-n)(n+1)$
	$m(n+1) - 2n$

Hence this is proved.

This rule has already been given in *Gaṇitasārasaṃgraha*and Mahāvīrācāryaclassifies it under *Vicitra Kuṭṭīkāra* problems (curious and interesting problems involving proportionate division). The rule and the example given by GK and GSS read almost verbatim.

Example - 5

कामुकाःपञ्चपरायस्त्रियस्तेषुचद्वौप्रियावप्रियास्त

त्रयस्तान्पृथक्।

त्वंप्रियोऽसीतिमेभाषमाणाऽद्भुतंकानिसत्यान्य

सत्यानिशीघ्रंवद॥

- Chapter II (Example 49)

"Two (2) Persons out of five (5), in love with another's wife, are liked by her and the others, disliked. She talks in wonderful language to each of them, separately, that only he is liked by her (and the others, disliked). Find out the number of her true and false statements separately".

Here $m = 5$ and $n = 3$.

Formula to find out the total number of true statements = $m^2 - m(n+1) + 2n$.

i.e. $5^2 - 5(3+1) + 2(3) = 11$.

Formula to find out the number of false statements = $m(n+1) - 2n$

i.e. $5(3+1) - 2(3) = 14$.

This can be presented using a table :

	A1	A2	A3	A4	A5	True Statements Total
A1	T	F	F	T	T	3
A2	F	T	F	T	T	3
A3	F	F	T	T	T	3
A4	F	F	F	F	T	1
A5	F	F	F	T	F	1
						11

A1, A2, A3	Liked by her
A4, A5	Not liked by her
T	True Statement
F	False Statement

First row refers to ‘if the lady tells the first person that he is liked by her’. This statement with reference to A1 is true, A2 is False, A3 is False, A4 and A5 are True.

Therefore with reference to the first person (A1) Number of True statements is 3 and with reference to the second person (A2) is 3, A3 is 3, A4 is 1 and A5 is 1 and so on.

Total number of True statements is 11 and it matches with answer derived using the formula.

Conclusion

Classification of fractions by various ancient Indian mathematical texts alone can be taken up as a separate study to understand the continuity of computational tradition of our ancient wisdom.

There are many such unique problems in the ancient Indian mathematical texts. Introducing the present generation of students to these types of problems will allay their fears about the complexity of the ancient works.

References

Datta, B. B. and Singh, A.N.1962.*History of Hindu Mathematics* (Vol.1-2),Bombay: Asia Publishing House.

Gaṇita Kaumudī of Nārāyaṇa Paṇḍita, Ed. by Padmakar Dvivedi, (two parts), The Princess of Wales, Saraswati, Bhavan Texts, No. 57, Benaras, 1936 and 1942.

Gaṇita Kaumudī of Nārāyaṇa Paṇḍita, English Translation With Notes : Parmanand Singh in *Gaṇita Bhāratī ,* 1998 – 2001 (Vol. 20 to 24).

Rangacharya M., 1912.*Gaṇitasarasangraha of*Mahāvīra, Madras : Government Press.

Shukla, 1971. Hindu Mathematics in the seventh century as found in Bhāskara I’s commentary on the *Āryabhaṭiya* (I), Gaṇita 22(1): 118.